An Analysis of Students' Difficulties in Solving PISA-like Mathematical Problems

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An Analysis of Students' Difficulties in Solving PISA-like Mathematical Problems

F Rahmawati and H Retnawati

Graduate School Yogyakarta State University, Yogyakarta, Indonesia

E-mail: 13709251062@student.uny.ac.id

Abstract. PISA (Programme for International Student Assessment) is one of international assessments of mathematics and science achievement that focuses on measuring knowledge and skill of 15-year-old students. It has acquired key knowledge and skills that are essential for full participation in modern societies. Indonesia has joined this program since 2000 but the result said the mathematical literacy of Indonesian students is lower than that of in other country. This study aims to explore students' difficulties in solving PISA-like mathematical problems, so the results of this study can be used as a consideration in the teaching and learning process. For this purpose, a test addressing PISA mathematics tasks were administered to 70 ninth graders from three junior high schools in the Province of Yogyakarta. Newman error analysis was performed to analyse students' incorrect responses in essay problems. The result of this study shows that students did not get significant difficulties on solving multiple choices problem, but there were difficulties in solving essays problems. The result of this study shows that students did error on reading, comprehension, transformation, process skill, and encoding.

1. Introduction

PISA (Programme for International Student Assessment) is assessment organized by OECD (Organization for Economic Co-operation and Development) held every three years since 2000. PISA measures knowledge and skill of 15-year-old students in mastering processes, understanding concepts, and students' ability in solving various types of problems. The aim of PISA is to measure what skills and competencies students have mastered and can apply in reading, mathematics, and science to real world contexts by age 15. [1, 2]

In each year when PISA is implemented, there is always one of the major domains that will be tested in details. The major domain in 2015 is science, as it was in 2006. Reading was the major domain in 2000 and 2009, and mathematics was the major domain in 2003 and 2012. The assessment areas in PISA 2015 were in science as the focus area, while reading, mathematics as well as collaborative problem solving as the minor areas. There were 510.000 students as representation of 28 million 15-year-old students from 65 countries participated that completed the PISA assessment in 2012. An additional 7 countries participated in 2015 [2].

Stacy said the key features of PISA are as policy orientation, with the major aim of informing educational policy and practice; the PISA concept of literacy with a foundation of assessment of literacy for reading, mathematics and science; its relevance to lifelong learning, so that assessment of knowledge

is supplemented by reports on motivation to learn, attitudes towards learning and learning strategies; its regularity, enabling countries to monitor improvements in educational outcomes in the light of other countries' performances on assessments held every three years; measurement of student performance alongside characteristics of students and schools, in order to explore some of the main features associated with educational success; breadth, with over 60 countries and economies participating by 2009, representing around 90% of the world economy [3].

There are three domains in PISA; reading literacy, scientific literacy and mathematical literacy. OECD defined mathematical literacy as students' ability to formulate, employ and interpret mathematics in various contexts. Mathematical literacy also includes mathematical reasoning and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena [2]. De Lange stated that mathematical literacy covers the following fields [4].

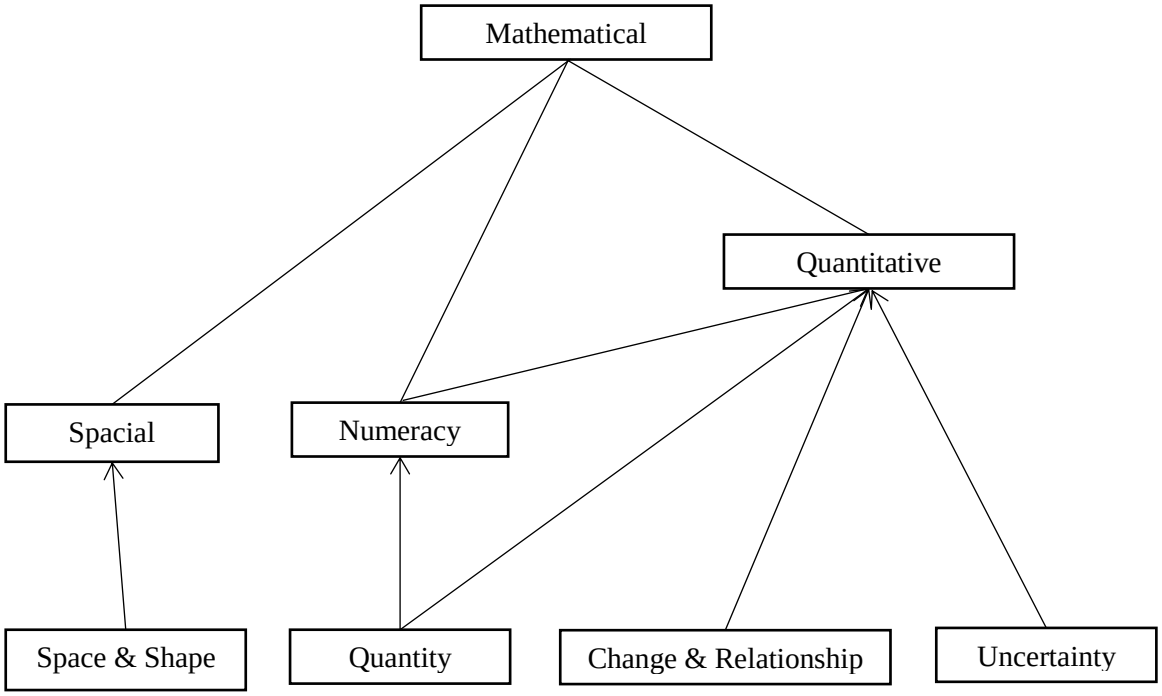


Figure 1. Tree structure mathematical literacy by De Lange

De Lange stated the OECD/PISA mathematical literacy domain is concerned with the capacities of students to analyze, reason, and communicate ideas effectively as they pose, formulate, solve and interpret mathematics in a variety of situations [4]. The assessment focuses on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms. In real-world settings, citizens regularly face situations when shopping, travelling, cooking, dealing with personal finances, etc. in which mathematical competencies would be of some help in clarifying or solving a problem.

Indonesia has joined PISA since 2000, but the mathematical literacy of Indonesian students is lower than the other country. It is because they are not accustomed to work on PISA-like mathematical problems [5]. Zulkardi stated that national examination problem in Indonesia is different with PISA problem. Indonesian national examination problem classified as low level problem in PISA problem [6]. Therefore, this research aims to reveal the difficulties often faced by ninth grade students in solving

PISA-like mathematical problems so the results of this study can be used as a consideration in the teaching and learning process.

Some related research that were used in this research are research conducted by Sumule et al in 2018 entitled Error Analysis of Indonesian Junior High School Student in Solving Space and Shape Content PISA Problem Using Newman Procedure. In results, comprehension and transformation errors appeared from students' performance in this research. The reasons are students was not able to identify the keywords in the question, wrote down what was known or given, specify formulas or device a plan [7]. Another study that became the reference for this research was research conducted by Mushlihah et al in 2018 entitled Analysis Problem Solving in Mathematical Using Theory Newman. The results of this study were 4.35% of students did reading error, 17.39% of students did comprehension error, 34.78% of students did transformation error, process skill errors of 23.91%, and 19.57% of students did encoding error [8].

2. Method

2.1. Participant

The participants of this study were 70 ninth graders from three junior high schools in Yogyakarta. The selection of these grades was in accordance with the target group of PISA study that is 15-year-old students.

2.2. Test

The objective of the research was to investigate students' difficulties in solving PISA-like problem. For this purpose, a test addressing PISA-like problem was administered. There were 30 problems, 9 multiple choices and 21 essays. In order to explore students' difficulties, the test included PISA's domain in mathematical literacy, namely:

- The mathematical processes that describe what individuals do to connect the context of the problem with mathematics and thus solve the problem, and the capabilities that underlie those processes. In OECD stated that mathematical process defined in three categories, formulating situation mathematically; employing mathematical concepts, facts, procedures and reasoning; and interpreting, applying and evaluating mathematical outcomes. These three processes are based on seven basic mathematical abilities namely communication; mathematics; representation; reasons and arguments; develop strategies to solve problems; use symbolic, formal and technical language and operations; use mathematical tools. All of these capabilities are based on the problem solver's detailed mathematical knowledge about individual topics [2].
- The mathematical content that is targeted for use in the assessment items. In OECD stated that mathematical content are four ideas (quantity, space and shape, change and relationships, and uncertainty and data) that are related to familiar curricular subjects, such as numbers, algebra and geometry, in overlapping and complex ways [2].
- The contexts in which the assessment items are located. In OECD stated that mathematical contexts are the settings in a students' world in which the problems are placed. The framework identifies four contexts: personal, educational, societal and scientific [2].

There were also five types of PISA problem that used in this study. Shiel stated the five types of PISA problem as [9]:

- Traditional multiple-choice items, in which the student selects a response from among several alternatives. The example of traditional multiple-choice problem can be seen in figure 2.

UNIT 9: SWEET MARTABAK

Here is a recipe for making sweet martabak for 400 grams of rice flour as the main ingredient, while the other ingredients with details are as follows.

Sugar	200 gram
Coconut Milk	700 cc
Salt	2 tsp

How much sugar is needed to make sweet martabak with the main ingredient of 1 kg of rice flour?

A. 500 gram C. 1000 gram
B. 800 gram D. 2000 gram

Figure 2. The Example of Traditional multiple-choice items.

- Complex multiple-choice items, in which the student chooses responses for a series of items. The example of complex multiple-choice problem can be seen in see figure 3.

UNIT 10: MATHEMATICS TEST

In the mathematics test, it is known that the mean is 58. The number of male students in the class is 12 and there are 21 female students.

Based on the information provided above, determine the truth value from the statements below:

Statement	True/False
If a student got 60 on his test, there must be another student who got 56.	True/False
Most of the student got 58 on their test	True/False
If the student score is sorted from lowest to highest, the median is 58.	True/False
50% of students in the class must get a score below 58 and 50% of other students must get a score above 58.	True/False

Explain your answer

Figure 3. The example of complex multiple-choice items.

- Closed-constructed response items, in which the answer is given in numeric or other form, and can be scored against precisely-defined criteria. The example of closed-constructed response items can be seen in figure 4.

UNIT 2 : CYCLING

Abi, Ardi, and Ari are cycling every Sunday. The diameter of Abi's wheel bike is 56 cm, the diameter of Ardi's wheel bike is 35 cm, the diameter of Ari wheel bike is 70 cm. If Abi and Ari paddle their bikes ten rounds full, how long does their distance? Give your answer in meters!

Figure 4. The example of closed-constructed response items.

- Short-response items, in which the student writes a brief answer to a question. Unlike closed-constructed response items, there may be a range of possible correct responses. The example of short-response items can be seen in figure 5.

UNIT 8: INTERNET VOUCHER

A variety of internet vouchers are offered as follows:

Variety	Price	Quota	Active Period	Bonus
Voucher A	Rp 130.000,00	12 GB	12 months	Phone credit Rp 3.000
Voucher B	Rp 65.000,00	6 GB	12 months	
Voucher C	Rp 42.000,00	4 GB	12 months	
Voucher D	Rp 35.000,00	3 GB	3 months	Phone credit Rp 1.000
Voucher E	Rp 21.500,00	2 GB	2 months	
Voucher F	Rp 11.000,00	1 GB	1 months	

Dina will buy an internet voucher for her mobile phone. Every month Dina always spend 1GB internet data package. Which internet voucher packages should Dina buy for an internet connection for 1 year at the lowest price? What is the price that Dina has to pay to buy an internet package for 1 year? Explain your answer!

Figure 5. The example of Short-response item.

- Open-constructed response items, in which the student provides a longer written response. There is usually a broad range of possible correct responses. Unlike other item types, the scoring of these questions typically requires significant judgement on the part of trained markers. The example of open-constructed response items can be seen in figure 6.

UNIT 12: DICE GAME

Two players each throw two dices. From the dice number that appears, the largest dice number is reduced by the smallest dice number. If the difference is 0, 1, or 2 then player A gets 1 point. However, if the difference is 3, 4 or 5 then player B gets 1 point. The game ends after 12 throws of the dice and the player with the most points is the winner.

Is the game fair? If the game is unfair, how can the rules be changed so that the game becomes fair?

Figure 6. The example of Open-constructed response items.

In multiple choices problems, there are three types of distractions namely misconception, careless and miscalculation.

2.3. Analysis

Students' difficulties was measured on the basis of students' responses to tasks with the above mentioned characteristics (see figure 1, figure 2, figure 3, figure 4, figure 5 and figure 6). For multiple choices problems, the analysis was by detecting which distractor was the students chose. For essay problems, students' incorrect and partially correct responses to such tasks were analyzed by using an analysis framework developed by Newman as follow [10]:

2.3.1. Reading Error. A reading error occurred when student failed on recognizing the written word or symbols that led the students failed in solving problems.

2.3.2. Comprehension Error. A comprehension error occurred when the pupil was able to read the question but failed to understand its requirement, thus causing him/her to error in or to fail at attempting problem-solution.

2.3.3. Transformation Error. A transformation error occurred when student was able to comprehend the question's requirement but failed to identify the proper mathematical operation or sequence of operation that led the students failed in solving problems.

2.3.4. Process Skill Error. A process skill error occurred when student was able to identify the proper mathematical operation or sequence of operation to solve the problem but failed to carry out the procedure correctly.

2.3.5. Encoding Error. An encoding error occurred when student have obtained the correct mathematical task but failed on providing an acceptable written form of the answer.

3. Result and Discussion

3.1. Multiple Choice Test

From 630 possible responses, 407 were correct (64.60%), 128 were missing (20.32%), and 95 were incorrect (15.08%). There were three errors in multiple choices test (see table 1)

Table 1. Multiple choices errors.

Types of error	Frequency	Percentage
Misconception	62	65,26%
Carelessness	21	22,11%
Miscalculation	12	12,63%
Total	95	100%

The purpose of this study was to investigate students' difficulties in solving PISA-like mathematical problems. Since 64.60% responses are correct, so it can be concluded that students did not find any fundamental difficulties in solving multiple choice PISA-like mathematical problems. The example of student's response that incorrect in misconception distraction can be seen on figure 7.

Problem:

A food company designed two sizes of instant soup cans as shown on the side. If each of these cans can contain as many soups, how high can the A can be made? Show your calculation

A. 9cm

B. 10 cm

C. 11.5 cm

D. 12.5 cm

Answer

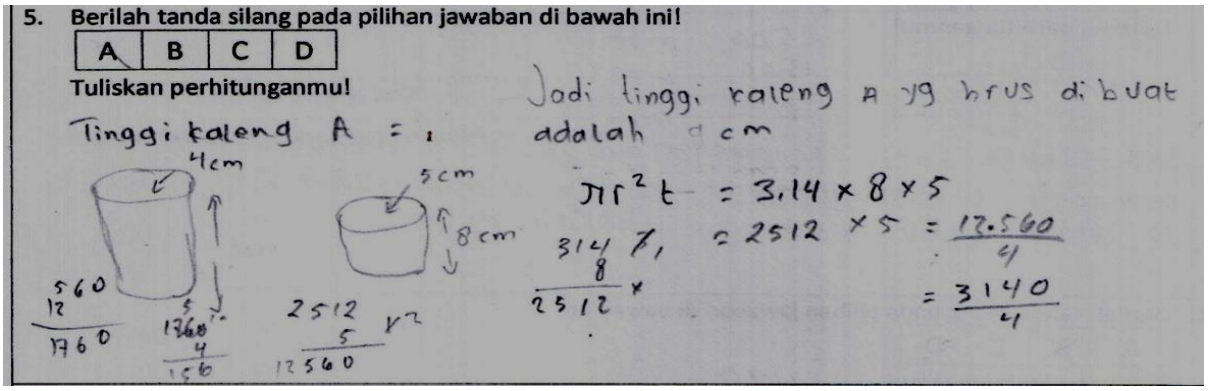


Figure 7. Example of misconception error.

From figure 7 we know that the student got wrong answer because he could not determine every elements given in can A. The correct answer for this problem was

$$V_A = V_B$$

$$\pi r_A{}^2 t_A = \pi r_B{}^2 t_B$$

$$4^2 t = 5^2 \times 8$$

$$t = \frac{200}{16} = 12.5cm$$

The example of student's response that incorrect in carelessness error distraction can be seen on figure 8.

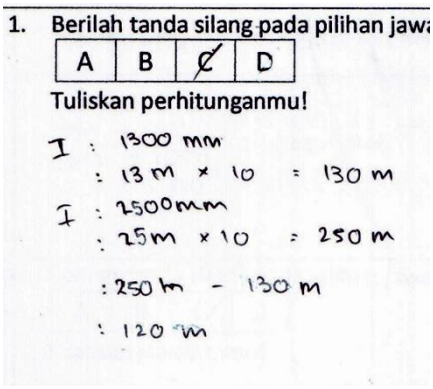
Problem	**Answer**
Three rollers are used to pave the way. The baby roller (roller I) has a wheel circumference of 1300 mm, the single drum roller (roller II) with a wheel circumference of 2100 mm, and a tandem roller (roller III) with a wheel circumference of 2500 mm. What is the difference in the distance of the asphalt using the I and III roller when the engine is operated as many as ten full rounds of wheels? Show your calculations! A. 12.000 m B. 1200 m C. 120 m D. 12 m	

Figure 8. Example of carelessness error.

From figure 8 we know that the student got wrong answer because he could not determine the correct changes from millimeters to meters. The correct answer for this problem was

$$(2500 \times 10)mm - (1300 \times 10)mm = 25000 - 13000 = 12000mm = 12m$$

The example of student's response that incorrect in miscalculation error distraction can be seen on figure 9.

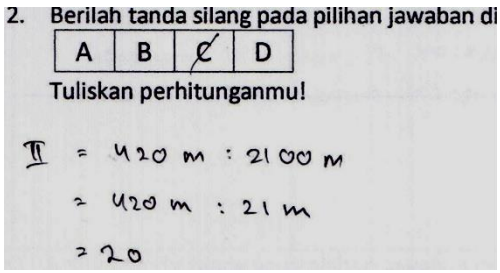
PROBLEM	**ANSWER**
Three rollers are used to pave the way. The baby roller (roller I) has a wheel circumference of 1300 mm, the single drum roller (roller II) with a wheel circumference of 2100 mm, and a tandem roller (roller III) with a wheel circumference of 2500 mm. How many wheel spin the roller II when used to pave 420 meters? Show your calculations! A. 200 C. 20 B. 168 D. 5	

Figure 9. Example of miscalculation error.

From figure 9 we know that the student got wrong answer because he could not determine the correct changes from millimeters to meters. The correct answer for this problem was

Circumference of the wheel = 2100 mm = 2,1 m

$$\text{Amount of wheel spin} = \frac{420}{2.1} = 200$$

3.2. Essay Test

Out of 1470 possible responses, 172 were correct (4.09%), 404 were partially correct (27.48%), 7 were incorrect (0.48%) and 887 were missing (60.34%). There were five types of errors that analyzed in this study (see table 2).

Table 2. Essay errors.

Types of error	Frequency	Percentage
Reading	172	10,98%
Comprehension	619	39,53%
Transformation	422	26,95%
Process skill	232	14,81%
Encoding	121	7,73%
Total	1566	100%

The example of student's response that incorrect in comprehension error can be seen on figure 10.

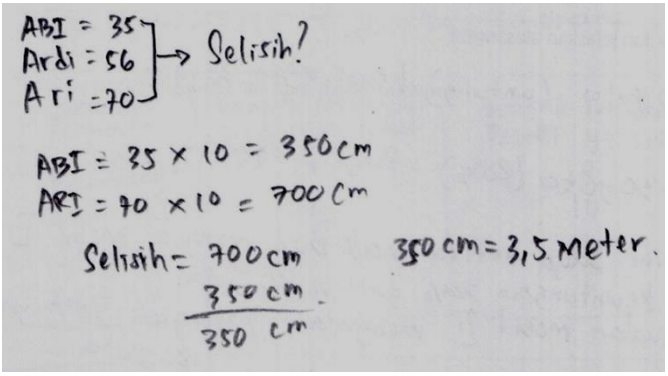
PROBLEM	**ANSWER**
Abi, Ardi, and Ari cycled together every Sunday. Abi's bicycle wheel diameter is 35 cm, Ardi's bicycle wheel diameter is 56 cm and Ari's bicycle wheel diameter is 70 cm. If Abi and Ari paddled their ten rounds full of wheels, what was the difference in the distance between the two bicycles? Give your answer in meters!	

Figure 10. Example of comprehension error.

From figure 10 we know that the student got wrong answer because he fail on determining the meaning of diameter as distance. The correct answer was

Distance = wheel circumference

Abi's wheel circumference $= \pi \cdot d = \frac{22}{7} \times 35 = 110cm$

The distance traveled by Abi after ten rounds is 11 m.

Ari's wheel cirumference $= \pi \cdot d = \frac{22}{7} \times 70 = 220cm$

The distance traveled by Ari after ten rounds is 22 m.

The distance between abi and ari after ten full turns of the wheel ish 22-11 = 11 meter.

The example of student's response that incorrect in comprehension error can be seen on figure 11

PROBLEM
Mr. Budi will sell his two cars then buy a new car. Both cars are sold at a price of 150 million each. From the proceeds of the sale, Mr. Budi made 20% of the sales of car I and lost 20% of cars II from the sale of cars II. From the above problems, does Mr. Budi experience profits or losses? Explain your reason!

ANSWER

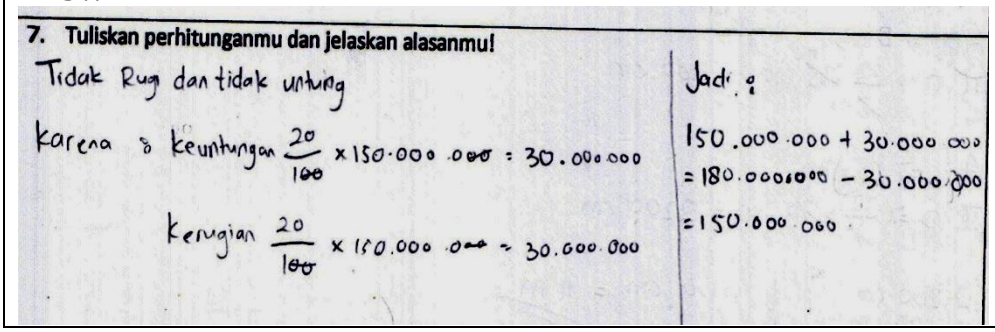


Figure 11. Example of transformation error.

From figure 11 we know that the student got wrong answer because he fail on using the formula of profit and loss so the conclusion was incorrect. The correct answer was

Car I	**Car II**
Selling price = Purchase price + profit	Selling price = Purchase price - loss
$150.000.000 = purchase + profit$	$150.000.000 = purchase - loss$
$150.000.000 = purchase + 20\% purchase$	$150.000.000 = purchase - \frac{1}{5} purchase$
$150.000.000 = purchase + \frac{1}{5} purchase$	$150.000.000 = \frac{4}{5} purchase$
$150.000.000 = \frac{6}{5} purchase$	$purchase = 187.000.000$
$purchase = 125.000.000$	$loss = 37.500.000$
$profit = 25.000.000$	

because the loss is greater than the profit, it can be concluded that Mr. Budi suffered a loss

Figure 12. Example of transformation error.

4. Conclusion

Identifying students' difficulties in solving mathematics problems is an essential aspect of determining the style of learning process in classroom. The results of this study shows students did not find fundamental difficulties in solving multiple choices PISA-like mathematical problem. But in solving essay PISA-like mathematical problem, only 4.09% students can solve the problem correctly. The result of analyzing incorrect and partially correct responses shows that there are 21.29% reading errors,

25.54% comprehension errors, 34.38% transformation errors, 19.14% processing errors, and 7.49% encoding errors.

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